# CHAPTER 2.

# *DOSIMETRIC PRINCIPLES, QUANTITIES AND UNITS*

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## 2.1. INTRODUCTION

Radiation measurements and investigations of radiation effects require various specifications of the radiation field at the point of interest. Radiation dosimetry deals with methods for a quantitative determination of energy deposited in a given medium by directly or indirectly ionizing radiations. A number of quantities and units have been defined for describing the radiation beam and the most commonly used dosimetric quantities and their units are defined below. A simplified discussion of cavity theory, the theory that deals with calculating the response of a dosimeter in a medium, is also given.

## 2.2. PHOTON FLUENCE AND ENERGY FLUENCE

The following quantities are used to describe a monoenergetic ionizing radiation beam: *particle fluence*, *energy fluence*, *particle fluence rate* and *energy fluence rate*. These quantities are usually used to describe photon beams and may also be used in describing charged particle beams.

- The particle fluence $\phi$ is the quotient $dN$ by $dA$, where $dN$ is the number of particles incident on a sphere of cross-sectional area $dA$

$$\phi = \frac{dN}{dA} \tag{2.1}$$

The unit of particle fluence is *particles*/cm$^2$. The use of a sphere of cross-sectional area $dA$ expresses in the simplest manner the fact that one considers an area $dA$ perpendicular to the direction of each particle and hence that particle fluence is independent of the incident angle of the radiation.

*Planar* particle fluence is the number of particles crossing a plane per unit area and hence depends on the angle of incidence of the particle beam.

- The energy fluence $\Psi$ is the quotient of *dE* by *dA*, where *dE* is the radiant energy incident on a sphere of cross-sectional area *dA*:

$$\Psi = \frac{dE}{dA} \tag{2.2}$$

The unit of energy fluence is J/m$^2$. Energy fluence can be calculated from particle fluence by using the following relationship:

$$\Psi = \frac{dN}{dA} E = \Phi E\,, \tag{2.3}$$

` where *E* is the energy of the particle and *dN* represents the number of particles with energy *E*.

- Almost all realistic photon or particle beams are polyenergetic and the above defined concepts need to be applied to such beams. The concepts of particle fluence spectrum and energy fluence spectrum replace the particle fluence and energy fluence, respectively. They are defined respectively as:

$$\varphi_E(E) \equiv \frac{d\varphi}{dE}(E) \tag{2.4}$$

and

$$\Psi_E(E) \equiv \frac{d\Psi}{dE}(E) = \frac{d\varphi}{dE}(E) \times E\,, \tag{2.5}$$

where $\varphi_E(E)$ and $\Psi_E(E)$ are shorthand notations for the particle fluence spectrum and the energy fluence spectrum, differential in energy *E*, respectively.

Figure 2.1. shows a photon fluence and an energy fluence spectrum generated by an orthovoltage x-ray unit with a $kV_p$ value of 250 kV and an added filtration of 1 mm Al and 1.8 mm Cu (target material: W; inherent filtration: 2 mm Be). The two spikes superimposed onto the continuous bremsstrrahlung spectrum represent the $K_\alpha$ and the $K_\beta$ characteristic x-ray lines produced in the tungsten target.

- The particle fluence rate φ is the quotient of $d\phi$ by *dt* where $d\phi$ is the increment of the fluence in the time interval *dt* :

$$\varphi = \frac{d\phi}{dt}\,, \tag{2.6}$$

with units of *particles*·m$^{-2}$·s$^{-1}$.

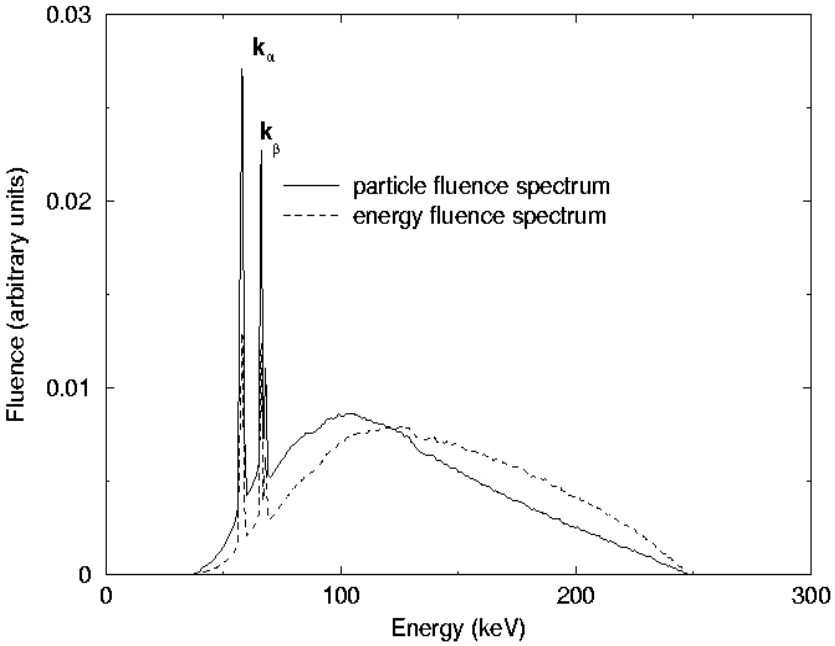


*FIG. 2.1. Photon fluence and energy fluence spectra at 1 m from the target of an x-ray machine with tube potential 250 kV and added filtration of 1 mm Al and 1.8 mm Cu (target material: W; inherent filtration 2 mm Be).*

- The energy fluence rate (also referred to as intensity) is the quotient of $d\psi$ by $dt$, where $d\psi$ is the increment of the energy fluence in the time interval $dt$:

$$\psi = \frac{d\Psi}{dt}, \tag{2.7}$$

The unit of energy fluence rate is $W/m^2$ or $J \cdot m^{-2} s^{-1}$.

## 2.3. KERMA

Kerma is an acronym for Kinetic Energy Released per unit MAss. It is a non-stochastic quantity applicable to indirectly ionizing radiations, such as photons and neutrons. It quantifies the average amount of energy *transferred* from the indirectly ionizing radiation to directly ionizing radiation without concerns to what happens after this transfer. In the discussion that follows we will limit ourselves to photons.

- Energy of photons is imparted to matter in a two-stage process. In the first stage, the photon radiation transfers energy to the secondary charged particles (electrons) through various photon interactions (photo-effect, Compton effect, pair production, etc). In the second stage, the charged particle transfers energy to the medium through atomic excitations and ionisations.

- In this context, the kerma is defined as the mean energy transferred from the indirectly ionizing radiation to charged particles (electrons) in the medium $d\overline{E}_{tr}$ per unit mass $dm$:

$$K = \frac{d\bar{E}_{\mathrm{tr}}}{dm} \quad . \tag{2.8}$$

The unit of kerma is joule per kilogram ($J \cdot kg^{-1}$). The special name for the unit of kerma is the gray (Gy), where 1 Gy = 1 $J \cdot kg^{-1}$.

## 2.4. CEMA

- Cema is the acronym for Converted Energy per unit MAss. It is a non-stochastic quantity applicable to directly ionizing radiations, such as electrons and protons.

- The cema $C$ is the quotient of $dE_c$ by $dm$, where $dE_c$ is the energy lost by charged particles, except secondary electrons, in electronic collisions in a mass $dm$ of a material:

$$C = \frac{dE_c}{dm} \quad . \tag{2.9}$$

The unit of cema is joule per kilogram ($J \cdot kg^{-1}$). The special name for the unit of cema is the gray (Gy).

## 2.5. ABSORBED DOSE

Absorbed dose is a non-stochastic quantity applicable to both indirectly and directly ionizing radiations. For indirectly ionizing radiations, energy is imparted to matter in a two step process. In the first step (resulting in kerma) the indirectly ionizing radiation transfers energy as kinetic energy to secondary charged particles. In the second step these charged particles transfer some of their kinetic energy to the medium (resulting in absorbed dose) and lose some of their energy in the form of bremsstrahlung losses.

- The absorbed dose is related to the stochastic quantity energy imparted. The absorbed dose is defined as the mean energy $\bar{\varepsilon}$ imparted by ionizing radiation to matter of mass $m$ in a finite volume $V$ by:

$$D = \frac{d\bar{\varepsilon}}{dm}. \tag{2.10}$$

- The energy imparted $\bar{\varepsilon}$ is the sum of all energy entering the volume of interest minus all energy leaving the volume, taking into account any mass-energy conversion within the volume. Pair production, for example, decreases the energy by 1.022 MeV, while electron-positron annihilation increases the energy by the same amount.

- Note that because the electrons travel in the medium and deposit energy along their tracks, this absorption of energy does not take place at the same location as the transfer of energy described by kerma. The unit of absorbed dose is joule per kilogram ($J \cdot kg^{-1}$). The special name for the unit of absorbed dose is the gray (Gy).

## 2.6. STOPPING POWER

Stopping powers are widely used in radiation dosimetry, but they are rarely measured and have to be calculated from theory. For electrons and positrons the Bethe theory is used to calculate stopping powers.

- The linear stopping power is defined as the expectation value of the rate of energy loss per unit path length $(dE/dx)$ of the charged particle. The mass stopping power is defined as the linear stopping power divided by the density of the absorbing medium. Division by the density of the absorbing medium almost eliminates the dependence of the mass stopping power on mass density except for the density effect discussed further below. Typical units for the linear and the mass stopping powers are MeV/cm and MeV·cm$^2$/g, respectively.

- Two types of stopping powers are known: *collision (ionisation)* resulting from interactions of charged particles with atomic orbital electrons and *radiative* resulting from interactions of charged particles with atomic nuclei.

- The unrestricted mass collision stopping power expresses the average rate of energy loss by a charged particle in all hard as well as soft collisions.

  - A *soft collision* occurs when a charged particle passes an atom at a considerable distance, *i.e.*, $b>>a$ where $b$ is the impact parameter and $a$ the atomic radius. The net effect of the collision is that a very small amount of energy is transferred to an atom of the absorbing medium in a single collision.
  - In a *hard collision* where $b \cong a$, a secondary electron (often referred to as a delta electron) with considerable energy is ejected and forms a separate track.
  - In the unrestricted mass collision stopping power the maximum energy transfer to an orbital electron allowed due to a hard collision is half of the kinetic energy of the electron (collision of indistinguishable particles), or the full kinetic energy of a positron (collision of distinguishable particles).

- The theory of the mass collision stopping power for heavy charged particles, electrons and positrons as a result of soft and hard collisions combines the Bethe theory for soft collisions with the stopping power as a result of energy transfers due to hard collisions. The result of this, for a heavy charged particle with mass $M$ and velocity $v$, where the energy transfer due to hard collisions is limited to $2m_e c^2\beta^2/(1-\beta^2)$, with $\beta = v/c$, is:

$$\frac{S_{\text{col}}}{\rho} = \frac{N_A Z}{A}\frac{\pi r_e^2 4m_e c^2}{\beta^2} z^2 \left[\ln\left(\frac{2m_e v^2}{I}\right) - \ln(1-\beta^2) - \beta^2\right], \tag{2.11}$$

where

$r_e$ is the classical electron radius (2.82 fm),
$z$ is the projectile charge in units of electron charge and
$I$ is the mean excitation potential of the medium.

- The mean excitation potential $I$ is a geometric mean value of all ionisation and excitation potentials of an atom of the absorbing material. Since binding effects influence the exact value of $I$, calculation models are often inadequate to accurately estimate its value. Hence, $I$ values are usually derived from measurements of stopping powers in heavy charged particle beams for which the effects of scattering in these measurements is minimal.

- For elemental materials $I$ varies approximately linearly with $Z$, with, on average, $I = 11.5\,Z$. For compounds $I$ is calculated assuming additivity of the collision stopping power taking into account the fraction by weight of each atom constituent in the compound.

- The following observations can be made from (Eq. (2.11)):

  - The mass stopping power does not depend on the projectile mass and is proportional to the inverse square of the projectile velocity. Note that the term $2m_o v^2$ under the logarithm has no relation with a kinetic energy of any of the particles involved in the collision process.
  - The mass stopping power gradually flattens to a broad minimum for kinetic energies $KE \cong 3m_e c^2$.
  - The leading factor $Z/A$ is responsible for a decrease of about 20% in stopping power from carbon to lead. The term $-\ln I$ causes a further decrease in stopping power with $Z$.
  - The square dependence on the projectile charge ($z^2$) causes heavy charged particles with double the charge to have four times the stopping power.

- For electrons and positrons, energy transfers due to soft collisions are combined with those due to hard collisions using the Moller (for electrons) and Bhabba (for positrons) cross-sections for free electrons. The complete mass collisional stopping power for electrons and positrons, according to ICRU Report 37, is:

$$\frac{S_{\text{col}}}{\rho} = \frac{N_A Z}{A}\frac{\pi r_o^2 2m_e c^2}{\beta^2}\left[\ln\left(KE/I\right)^2 + \ln(1+\tau/2) + F^{\pm}(\tau) - \delta\right]$$

with $F^-$ given for electrons as

$$F^-(\tau) = (1-\beta^2)[1+\tau^2/8-(2\tau+1)\ln 2] \qquad (2.12)$$

and $F^+$ given for positrons as

$$F^+(\tau) = 2\ln 2 - (\beta^2/12)[23+14/(\tau+2)+10/(\tau+2)^2+4/(\tau+2)^3].$$

In this equation, $\tau = KE/m_e c^2$ and $\beta = v/c$.

- $\delta$ is the density effect correction that accounts for the fact that the effective Coulomb force exerted on a fast charged particle by atoms that are distant from the particle track is reduced as a result of the polarization of the medium caused by the charged particle. The density effect affects the soft collision component of the stopping power. It plays a significant role in the values of ratios of stopping powers of a dense material to a non-dense material (such as, for example, water to air) and various models for it have been developed.

- The mass radiative stopping power is the rate of energy loss by electrons or positrons that results in the production of bremsstrahlung. The Bethe-Heitler theory leads to the following formula for the mass radiative stopping power:

$$\frac{S_{\text{rad}}}{\rho} = \sigma_{\text{o}} \frac{N_{\text{A}} Z^2}{A} (KE + m_{\text{e}}c^2) \overline{B}_{\text{r}} \tag{2.13}$$

where

$\sigma_{\text{o}} = \alpha \left( e^2 / (4\pi\varepsilon_{\text{o}} m_{\text{e}} c^2) \right)^2 = 5.80 \times 10^{-28}$ cm$^2$/atom where $\alpha$ is the fine structure constant, and $\overline{B}_{\text{r}}$ is a function of $Z$ and $KE$, varying between 5.33 and 15 for energies in the range from less than 0.5 MeV to 100 MeV.

This factor together with the increase of the radiative stopping power proportional with $KE$ is responsible for the increase in total stopping power at energies above 2 MeV as depicted in Fig. 2.2. Note that the $Z^2$ dependence of the mass radiative stopping power in contrast to the $Z$ dependence of the mass collision stopping power makes this mode of energy loss more prominent in high-$Z$ materials.

- The concept of restricted mass collision stopping power is introduced to calculate the energy transferred to a localized region of interest. By limiting the energy transfer to secondary charged (delta) particles to a threshold (often denoted as $\Delta$), highly energetic secondary particles are allowed to escape the region of interest.

- The restricted stopping power is lower than the unrestricted stopping power. The choice of the energy threshold depends on the problem at hand. For problems involving ionisation chambers a frequently used threshold value is 10 keV (the range of a 10 keV electron in air is on the order of 2 mm). For microdosimetric quantities one usually takes 100 eV as a reasonable threshold value.

- The restricted linear collision stopping power (also referred to as linear energy transfer) $L_{\Delta}$ of a material, for charged particles, is the quotient of $dE_{\Delta}$ by $d\ell$, where $dE_{\Delta}$ is the energy lost by a charged particle due to soft and hard collisions in traversing a distance $d\ell$ minus the total kinetic energy of the charged particles released with kinetic energies in excess of $\Delta$:

$$L_{\Delta} = dE_{\Delta} / d\ell \,. \tag{2.14}$$

The restricted mass collision stopping power is the restricted linear collision stopping power divided by the density of the material.

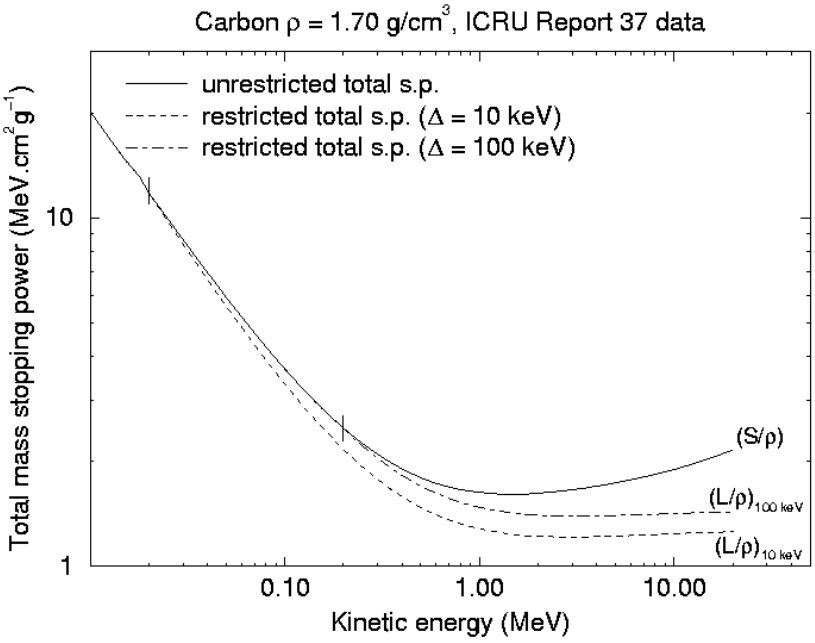


*FIG. 2.2. Unrestricted $S/\rho$ and restricted ($(L/\rho)_\Delta$ with $\Delta$ = 10 keV and 100 keV) total mass stopping powers for carbon, based on data published in the ICRU Report 37. Vertical lines indicate the points at which restricted and unrestricted mass stopping powers begin to diverge as the kinetic energy increases.*

- As the threshold for maximum energy transfer in the restricted stopping power increases, the restricted mass stopping power tends to the unrestricted mass stopping power for $\Delta \rightarrow KE/2$. Note also that since energy transfers to secondary electrons are limited to $KE/2$, unrestricted and restricted electron mass stopping powers are identical for kinetic energies lower than or equal to $2\Delta$. This is indicated in Fig. 2.2. with vertical lines at 20 keV and 200 keV.

- The total mass stopping power is the sum of the collision mass stopping power and the radiative mass stopping power. Figure 2.2. shows the total unrestricted and restricted ($\Delta$ = 10 keV, 100 keV) electron mass stopping powers for carbon based on the data in the ICRU Report 37.

## 2.7. RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.1. Energy fluence and kerma (photons)

The energy transferred to electrons by photons can be expended in two distinct ways:

(1) through collision interactions (soft collisions, hard collisions),
(2) through radiative interactions (bremsstrahlung, electron-positron annihilation).

Therefore, the total kerma is usually divided into two components: the *collision kerma* $K_{col}$ and the *radiative kerma* $K_{rad}$.

- *Collision kerma* $K_{\text{col}}$ is that part of kerma that leads to the production of electrons that dissipate their energy as ionisation in or near the electron tracks in the medium, and is the result of Coulomb-force interactions with atomic electrons. Thus, the collision kerma is the expectation value of the net energy transferred to charged particles per unit mass at the point of interest, excluding both the radiative energy loss and energy passed from one charged particle to another.

- *Radiative kerma* $K_{\text{rad}}$ is that part of kerma that leads to the production of bremsstrahlung as the secondary charged particles are decelerated in the medium. It is the result of Coulomb field interactions between the charged particle and the atomic nuclei.

- The total kerma $K$ is thus given by the following equation:

$$K = K_{\text{col}} + K_{\text{rad}} \,. \tag{2.15}$$

The average fraction of the energy transferred to electrons that is lost through radiative processes is represented by a factor referred to as the bremstrahlung fraction $\bar{g}$ . Hence, the fraction lost through collisions is (1- $\bar{g}$ ).

- A frequently used relation between collision kerma $K_{\text{col}}$ and total kerma K may be written as follows:

$$K_{\text{col}} = K(1-\bar{g}) \quad . \tag{2.16}$$

- For monoenergetic photons the collision kerma $K_{col}$ at a point in a medium is related to the energy fluence $\Psi$ at that point in the medium by the following equation:

$$K_{\text{col}} = \Psi \left( \frac{\mu_{\text{en}}}{\rho} \right), \tag{2.17}$$

where $(\mu_{\text{en}} / \rho)$ is the mass-energy absorption coefficient for the monoenergetic photons in the medium.

- For polyenergetic beams, a formally similar relation exists, but use is made of spectrum-averaged quantities. If a photon energy fluence spectrum $\Psi_{\text{E}}(E)$ is present at the point of interest, the collision kerma at that point is obtained as follows:

$$K_{\text{col}} = \int_{o}^{E_{\max}} \Psi_{\text{E}}(E) \left( \frac{\mu_{\text{en}}}{\rho} \right) \text{d}E = \Psi \left( \frac{\bar{\mu}_{\text{en}}}{\rho} \right) \quad . \tag{2.18}$$

In Eq. (2.18) $\Psi = \int_0^{E_{\max}} \Psi_E(E)dE$ stands for the total (integrated) energy fluence, and $\left(\frac{\bar{\mu}_{en}}{\rho}\right) = \frac{1}{\Psi}\int_0^{E_{\max}} \Psi_E(E)\frac{\mu_{en}}{\rho}(E)dE$ is a shorthand notation for the mass energy absorption coefficient for the medium averaged over the energy fluence spectrum.

- For mono-energetic photons the total kerma $K$ at a point in a medium is related to the energy fluence $\Psi$ in the medium by the following equation:

$$K = \Psi \left(\frac{\mu_{tr}}{\rho}\right), \tag{2.19}$$

where $(\mu_{tr}/\rho)$ is the mass-energy transfer coefficient of the medium for the given monoenergetic photon beam. For poly-energetic beams, similarly as above, spectrum-averaged mass-energy transfer coefficients can be used in conjunction with total energy fluence to obtain the total kerma.

- Note that, using Eq. 2.17, one can obtain the frequently used relation between collision kerma in two different materials, material 1 and material 2, as follows:

$$\frac{K_{col,2}}{K_{col,1}} = \frac{\Psi_2\left(\frac{\bar{\mu}_{en}}{\rho}\right)_2}{\Psi_1\left(\frac{\bar{\mu}_{en}}{\rho}\right)_1} \equiv (\Psi)_1^2\left(\frac{\bar{\mu}_{en}}{\rho}\right)_1^2 . \tag{2.20}$$

This equation is often used in circumstances where the fluence ratio $(\Psi)_1^2$ can be assumed unity through a proper scaling of dimensions (the scaling theorem), or for very similar materials.

### 2.7.2. Fluence and dose (electrons)

- Under the conditions that (1) radiative photons escape the volume of interest and (2) secondary electrons are absorbed on the spot (or there is charged particle equilibrium of secondary electrons), the absorbed dose to medium $D_{med}$ is related to the electron fluence $\phi_{med}$ in the medium as follows:

$$D_{med} = \phi_{med}\left(\frac{S_{col}}{\rho}\right)_{med}, \tag{2.21}$$

where $(S_{col}/\rho)_{med}$ is the unrestricted mass collision stopping power of the medium at the energy of the electron.

- Because of electron slowing down, even for a mono-energetic starting electron, there is always a primary electron fluence spectrum in the medium which is differential in energy and can be denoted by $\phi_{med,E}(E)$.

In this case, the absorbed dose to the medium can be obtained by an integration of Eq. 2.20:

$$D_{med} = \int_{o}^{E_{max}} \phi_{med,E}(E) \left(\frac{S_{col}}{\rho}\right)_{med}(E)\, dE = \phi_{med} \left(\frac{\bar{S}_{col}}{\rho}\right)_{med} . \quad (2.22)$$

The right hand side of Eq. (2.21) shows that absorbed dose can be calculated using a formally similar equation as Eq. (2.20) by making use of spectrum-averaged collision stopping power and total fluence.

- Based on Eq. (2.22) and under the same assumptions, for two media, $med_1$ and $med_2$, the ratio of absorbed doses can be calculated as:

$$\frac{D_{med_2}}{D_{med_1}} = (\phi)_{med_2,med_1} \cdot \left(\frac{\bar{S}_{col}}{\rho}\right)_{med_2,med_1} , \quad (2.23)$$

where the shorthand notations $(\phi)_{med_2,med_1}$ and $\left(\frac{\bar{S}_{col}}{\rho}\right)_{med_2,med_1}$ are being used for the ratio of the electron fluences (often referred to as the electron fluence ratio) and the collision stopping powers in the media $med_1$ and $med_2$, respectively.

- The full, realistic electron fluence spectrum consists of primary charged particles that, for example, are the result of a poly-energetic photon beam interacting in the medium. These primary charged particles are slowed down and result in secondary particle fluence.

### 2.7.3. Kerma and dose (charged particle equilibrium)

Generally, the transfer of energy (kerma) from the photon beam to charged particles at a particular location does not lead to the absorption of energy by the medium (absorbed dose) at the same location. This is due to the non-zero (finite) range of the secondary electrons released through photon interactions.

- Since radiative photons mostly escape from the volume of interest, one relates absorbed dose usually to collision kerma. In general, however, the ratio of dose and collision kerma is often denoted as:

$$\beta = D / K_{col} . \quad (2.24)$$

If radiative photons escape the volume of interest, an assumption is made that $\beta \approx 1$.

- Figure 2.3 illustrates the relationship between collision kerma and absorbed dose under build-up conditions; under conditions of charged particle equilibrium (CPE) in part (a) and under conditions of transient charged particle equilibrium (TCPE) in part (b).

As a high energy photon beam penetrates the medium, collision kerma is maximal at the surface of the irradiated material because photon fluence is greatest at the surface. Initially, the charged particle fluence, and hence the absorbed dose, increases as a function of depth until the depth of dose maximum $z_{max}$ is attained.

- If there were no photon attenuation or scattering in the medium, but yet production of electrons, a hypothetical situation, as illustrated in Fig. 2.3(a), would occur: the build-up region (with $\beta < 1$) is followed by a region of complete CPE where $D = K_{col}$, *i.e.*, $\beta = 1$.

- In the more realistic situation, however, due to photon attenuation and scattering in the medium, a region of TCPE occurs (Fig. 2.3(b)) where there exists an essentially constant relation between collision kerma and absorbed dose. This relation is practically constant since, in high-energy photon beams, the average energy of the generated electrons and hence their range does not change appreciably with depth in the medium.

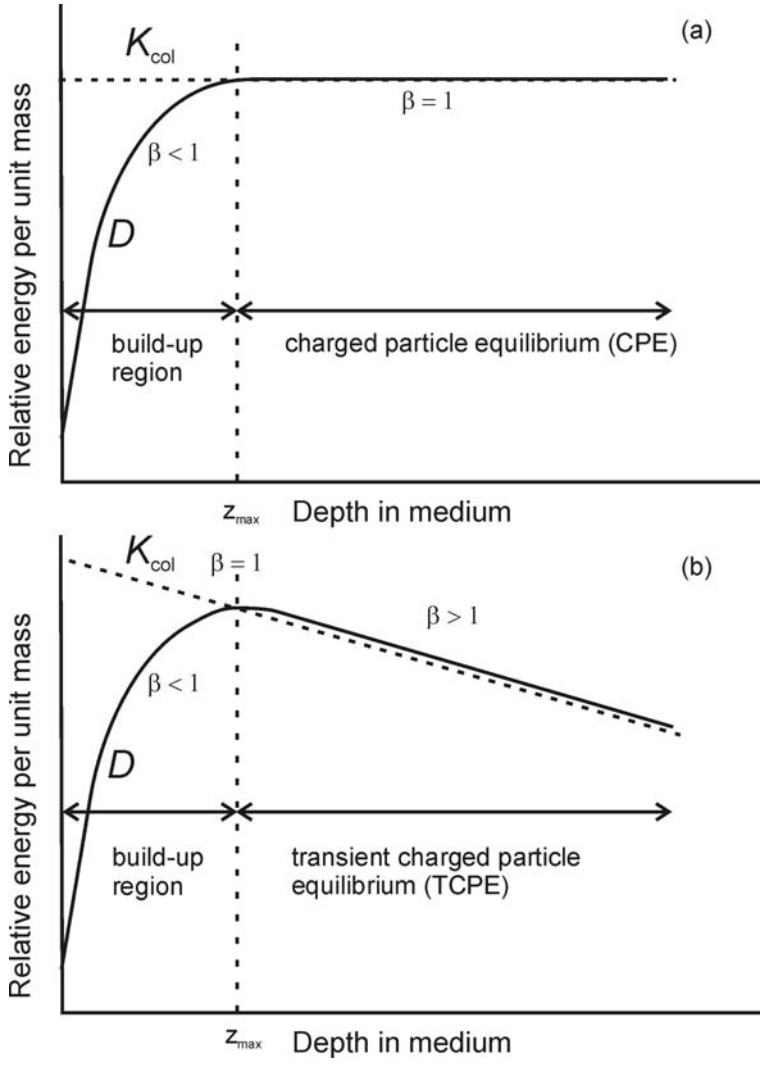


*FIG. 2.3. Collision kerma and absorbed dose as a function of depth in a medium, irradiated by a high-energy photon beam.*

- In the special case where true charged particle equilibrium exists (at the depth of maximum dose in the medium), the relation between absorbed dose $D$ and total kerma $K$ is given by:

$$D = K_{\text{col}} = K(1-\bar{g}) \quad , \tag{2.25}$$

where $\bar{g}$ is the bremsstrahlung fraction, depending on the electron kinetic energy; the higher the energy, the larger is $\bar{g}$. The bremsstrahlung fraction also depends on the material considered, with higher values of $\bar{g}$ for higher $Z$ materials. For electrons produced by cobalt-60 gamma rays in air the bremsstrahlung fraction equals to 0.0032.

- The build-up of absorbed dose is responsible for the skin sparing effect in the case of high-energy photon beams. However, in practice the surface dose is small but does not equal zero because of the electron contamination in the beam due to photon interactions in the media upstream from the phantom or due to charged particles generated in the accelerator head and beam modifying devices.

### 2.7.4. Collision kerma and exposure

- Exposure $X$ is the quotient of $dQ$ by $dm$, where $dQ$ is the absolute value of the total charge of the ions of one sign produced in air when all the electrons and positrons liberated or created by photons in mass $dm$ of air are completely stopped in air:

$$X = \frac{dQ}{dm} \quad . \tag{2.26}$$

The unit of exposure is coulomb per kilogram (C/kg). The special unit used for exposure is the roentgen $R$, where 1 R = 2.58 x $10^{-4}$ C/kg. In the SI system of units, roentgen is no longer used and the unit of exposure is simply $2.58 \times 10^{-4}$ C/kg of air.

- The average energy expended in air per ion pair formed $W_{air}$ is the quotient of $E$ by $N$, where $N$ is the mean number of ion pairs formed when the initial kinetic energy $E$ of a charged particle is completely dissipated in air:

$$W_{\text{air}} = \frac{E}{N} \quad . \tag{2.27}$$

The current best estimate for the average value of $W_{air}$ is 33.97 eV/*ion pair* or $33.97 \times 1.602 \times 10^{19}$ J/*ion pair*:

$$\frac{W_{\text{air}}}{e} = \frac{33.97 \ \left(\text{eV}/\textit{ion pair}\right) \times 1.602 \times 10^{-19} \left(\text{J}/\text{eV}\right)}{1.602 \times 10^{-19} \left(\text{C}/\textit{ion pair}\right)} = 33.97 \text{ J/C} \, . \tag{2.28}$$

- Multiplying the collision kerma by $\left(e/W_{\text{air}}\right)$, the number of coulombs of charge created per joule of energy deposited, gives the charge created per unit mass of air or exposure:

$$X = \left(K_{\text{col}}\right)_{\text{air}} \left(\frac{e}{W_{\text{air}}}\right). \tag{2.29}$$

- The relationship between total kerma and exposure is obtained by combining Eq. 2.25 and Eq. 2.29 to get:

$$K_{\text{air}} = X \left(\frac{W_{\text{air}}}{e}\right) \frac{1}{1-\bar{g}}. \tag{2.30}$$

## 2.8 CAVITY THEORY

In order to measure the absorbed dose in a medium, it is necessary to introduce a radiation sensitive device (dosimeter) into the medium. Generally, the sensitive medium of the dosimeter will not be of the same material as the medium in which it is embedded. Cavity theory relates the absorbed dose in the dosimeter sensitive medium (cavity) to the absorbed dose in the surrounding medium containing the cavity. Cavity sizes are referred to as small, intermediate or large in comparison with the ranges of secondary charged particles produced by photons in the cavity medium. If, for example, the range of charged particles (electrons) is much larger than the cavity dimensions, the cavity is regarded as small. Various cavity theories for photon beams have been developed depending on the size of the cavity, such as the Bragg-Gray and Spencer-Attix theories for small cavities and the Burlin theory for cavities of intermediate sizes.

### 2.8.1. The Bragg-Gray cavity theory

The Bragg-Gray (B-G) cavity theory was the first cavity theory developed to provide a relationship between absorbed dose in a dosimeter and the absorbed dose in the medium containing the dosimeter.

- The conditions for application of the Bragg-Gray cavity theory are:

  (1) the cavity must be small when compared with the range of charged particles incident on it so that its presence does not perturb the fluence of charged particles in the medium;
  (2) the absorbed dose in the cavity is deposited solely by charged particles crossing it, *i.e.*, photon interactions in the cavity are assumed negligible and thus ignored.

  The result of condition (1) is that the electron fluences in Eq. (2.22) are the same and equal to the equilibrium fluence established in the surrounding medium. This condition can only be valid in regions of CPE or TCPE. In addition, the presence of a cavity always causes some degree of fluence perturbation that requires the introduction of a fluence perturbation correction factor.

Condition (2) implies that all electrons depositing the dose inside the cavity are produced outside the cavity and completely cross the cavity. Therefore, no secondary electrons are produced inside the cavity and no electrons stop within the cavity.

- Under these two conditions, according to the Bragg-Gray cavity theory, the dose to the medium $D_{med}$ is related to the dose in the cavity $D_{cav}$ as follows:

$$D_{med} = D_{cav} \left( \frac{\overline{S}}{\rho} \right)_{med,cav}, \tag{2.31}$$

where $(\overline{S} / \rho)_{med,cav}$ is the ratio of the average unrestricted mass collision stopping powers of the medium and cavity. The use of unrestricted stopping powers rules out the production of secondary charged particles (or delta electrons) in the cavity and the medium.

- Although the cavity size is not explicitly taken into account in the Bragg-Gray cavity theory, the fulfillment of the two Bragg-Gray conditions will depend on the cavity size which is based on the range of the electrons in the cavity medium, the cavity medium, and electron energy. A cavity that qualifies as a Bragg-Gray cavity for high energy photon beams, for example, may not behave as a Bragg-Gray cavity in a medium-energy or low-energy x-ray beam.

### 2.8.2. The Spencer-Attix cavity theory

- The Bragg-Gray cavity theory does not take into account the creation of secondary (delta) electrons generated as a result of the slowing down of the primary electrons in the sensitive volume of the dosimeter. The Spencer-Attix (S-A) cavity theory is a more general formulation that accounts for the creation of these electrons which have sufficient energy to produce further ionisation on their own account. Some of these electrons released in the gas cavity would have sufficient energy to escape from the cavity carrying some of their energy with them. This reduces the energy absorbed in the cavity and requires modification to the stopping power of the gas. The Spencer-Attix cavity theory operates under the two Bragg-Gray conditions, however, these conditions now even apply to the secondary particle fluence in addition to the primary charged particle fluence.

- The secondary electron fluence in the Spencer-Attix theory is divided into two components based on a user-defined energy threshold $\Delta$. Secondary electrons with kinetic energies $KE$ less than $\Delta$ are considered slow electrons that deposit their energy locally; secondary electrons with energies larger than or equal to $\Delta$ are considered "fast" (slowing down) electrons and are part of the electron spectrum. Consequently, this spectrum has a low energy threshold of $\Delta$ and a high-energy threshold of $KE_o$ where $KE_o$ represents the initial electron kinetic energy. Since the lowest energy in the spectrum is $\Delta$, the maximum energy loss of a fast electron with kinetic energy $KE$ larger than or equal to $2\Delta$ cannot be larger than $\Delta$ and the maximum energy loss of a fast electron with kinetic energy less than $2\Delta$ cannot be larger than $KE/2$ (where $\Delta \leq KE < 2\Delta$).

- The energy deposition must be calculated as the product of $L_{\Delta}(KE)/\rho$, the restricted collision stopping power with threshold $\Delta$, and the fast electron fluence with electrons ranging in energy from $\Delta$ to $KE_{\mathrm{o}}$

- Because of the Bragg-Gray condition which stipulates that there must not be electron production in the cavity, the electrons with energy $\Delta$ must be capable of crossing the cavity. Hence, the threshold value $\Delta$ is related to the cavity size and is defined as the energy of the electron with range equal to the mean chord length across the cavity.

- The Spencer-Attix relation between the dose to the medium and the dose in the cavity is thus written as:

$$D_{\mathrm{med}} / D_{\mathrm{cav}} = s_{\mathrm{med,cav}} \ , \tag{2.32}$$

where $s_{\mathrm{med,cav}}$ is the ratio of the mean restricted mass collision stopping powers from the medium to the cavity.

Using the medium electron fluence spectrum $\Phi_{KE}^{med,\delta}\left(KE\right)$, the full expression is:

$$s_{\mathrm{med,cav}} = \frac{\int_{\Delta}^{KE_{\mathrm{o}}} \Phi_{KE}^{\mathrm{med},\delta}\left(KE\right)\left(L_{\Delta,med}/\rho\right)d(KE) + TE_{\mathrm{med}}}{\int_{\Delta}^{KE_{\mathrm{o}}} \Phi_{KE}^{\mathrm{med},\delta}\left(KE\right)\left(L_{\Delta,cav}/\rho\right)d(KE) + TE_{\mathrm{cav}}}. \tag{2.33}$$

The terms $TE_{\mathrm{med}}$ and $TE_{\mathrm{cav}}$ are called the track-end terms and account for a part of the energy deposited by electrons with initial kinetic energies between $\Delta$ and $2\Delta$. These electrons can have an energy loss that brings their kinetic energy to lower than $\Delta$. Their residual energy after such events should be deposited on the spot and these electrons are removed from the spectrum. The track-end terms are approximated by Nahum as:

$$TE_{\mathrm{med}} = \Phi_{\mathrm{KE}}^{\mathrm{med},\delta}\left(\Delta\right)\frac{S_{\mathrm{med}}\left(\Delta\right)}{\rho}\Delta, \tag{2.34}$$

and

$$TE_{\mathrm{cav}} = \Phi_{\mathrm{KE}}^{med,\delta}\left(\Delta\right)\frac{S_{\mathrm{cav}}\left(\Delta\right)}{\rho}\Delta. \tag{2.35}$$

Note that the unrestricted collision stopping powers can be used here because the maximum energy transfer for an electron with energy less than $2\Delta$ is less than $\Delta$.

- Monte Carlo calculations showed that the difference between the Spencer-Attix and Bragg-Gray cavity theories is non-negligible yet generally not very significant. Since collision stopping powers for different media show similar trends as a function of particle energy, their ratio for the two media is a very slowly varying function with energy.

- For a typical ionisation chamber used in water, the energy dependence of the stopping power ratio water-to-air arises from the difference in the density effect correction between the two materials.

### 2.8.3. Considerations in the application of cavity theory to ionisation chamber calibration and dosimetry protocols

- A dosimeter can be defined generally as any device that is capable of providing a reading that is a measure of the average absorbed dose deposited in its (dosimeter) sensitive volume by ionizing radiation. A dosimeter can generally be considered as consisting of a sensitive volume filled with a given medium, surrounded by a wall of another medium.

- In the context of cavity theories, the sensitive volume of the dosimeter can be identified as the "cavity", which may contain a gaseous, liquid or solid medium. Gas is often used as the sensitive medium, since it allows a relatively simple electrical means for collection of charges released in the sensitive medium by radiation.

- The medium surrounding the cavity of an ionisation chamber depends on the situation in which the device is used. In an older approach, the wall (often supplemented with a build-up cap) serves as the build-up medium and the Bragg-Gray theory provides a relationship between the dose in the gas and the dose in the wall. This situation is referred to as a *thick-walled ionisation chamber* and forms the basis of cavity chamber-based *air-kerma in air* standards and of the $C_\lambda$ -based dosimetry protocols of the 1970s. If, however, the chamber is used in a phantom without a build-up material, since typical wall thicknesses are much thinner than the range of the secondary electrons, the proportion of the cavity dose due to electrons generated in the phantom greatly exceeds the dose contribution from the wall and hence, the phantom medium serves as the medium and the wall is treated as a perturbation to this concept.

- In the case of a thick-walled ionisation chamber in a high energy photon beam, the wall thickness must be greater than the range of secondary electrons in the wall material to ensure that the electrons that cross the cavity arise in the wall and not in the medium. The Bragg-Gray cavity equation then relates the dose in the cavity to the dose in the wall of the chamber. By assuming that (1) the absorbed dose is the same as collision kerma and further assuming that (2) the photon fluence is not perturbed by the presence of the chamber, the dose in the medium is related to the dose in the wall by means of a ratio of the mass energy absorption coefficients of the medium and the wall $\left(\bar{\mu}_{\text{en}} / \rho\right)_{\text{med,wall}}$. The dose to the cavity gas is related to the ionisation produced in the cavity as follows:

$$D_{\text{gas}} = \frac{Q}{m}\left(\frac{W_{\text{gas}}}{e}\right) \quad , \tag{2.36}$$

where $Q$ is the charge (of either sign) produced in the cavity and $m$ is the mass of the gas in the cavity.

- Spencer-Attix cavity theory can be used to calculate the dose in the medium as:

(2.37)

$$D_{\text{med}} = D_{\text{wall}} \left( \frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{med,wall}} = D_{\text{gas}} s_{\text{wall,gas}} \left( \frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{med,wall}} = \frac{Q}{m} \left( \frac{W_{\text{gas}}}{e} \right) s_{\text{wall,gas}} \left( \frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{med,wall}},$$

where $s_{\text{wall,gas}}$ is the ratio of restricted mass collision stopping powers for cavity wall and gas with threshold $\Delta$. In practice, there are additional correction factors associated with Eq. (2.36) to satisfy the assumptions (1) and (2) made above.

A similar equation to Eq. (2.36) is used for *air-kerma in air* calibrations, however, here the quantity of interest is not the dose to the medium rather it is the *air-kerma in-air*. In this case, a substantial wall correction is introduced to ensure the presence of complete CPE in the wall to satisfy assumption (1) above.

- In the case of a thin-walled ionisation chamber in a high energy photon or electron beam, the wall, the cavity and the central electrode are treated as a perturbation to the medium fluence and the equation now involves the ratio of restricted collision stopping powers medium to gas as:

$$D_{\text{med}} = \frac{Q}{m} \left( \frac{W_{\text{gas}}}{e} \right) s_{\text{med,gas}} p_{\text{fl}} \, p_{\text{dis}} \, p_{\text{wall}} \, p_{\text{cel}}, \tag{2.38}$$

where

$p_{\text{fl}}$ is the electron fluence perturbation correction factor;
$p_{\text{dis}}$ is the correction factor for displacement of the effective measurement point
$p_{\text{wall}}$ is the wall correction factor, and
$p_{\text{cel}}$ is the correction factor for the central electrode.

Values for these multiplicative correction factors are summarized for photon and electron beams in typical dosimetry protocols (see Section 9.7 for details).

### 2.8.4. Large cavities in photon beams

A large cavity is a cavity with dimensions such that the dose contribution made by electrons inside the cavity originating from photon interactions outside the cavity can be ignored when compared with the contribution of electrons created by photon interactions within the cavity.

For a large cavity the ratio of dose cavity to medium is calculated as the ratio of the collision kerma in the cavity to the medium and is therefore equal to the ratio of the average mass-energy absorption coefficients, cavity to medium:

$$\frac{D_{\text{gas}}}{D_{\text{med}}} = \left( \frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{gas,med}}, \tag{2.39}$$

where the mass-energy absorption coefficients have been averaged over the photon fluence spectra in the cavity gas (numerator) and in the medium (denominator).

### 2.8.5. Burlin cavity theory for photon beams

Burlin extended the Bragg-Gray and Spencer-Attix cavity theories to cavities of intermediate dimensions by introducing, on a purely phenomenological basis, the large cavity limit to the Spencer-Attix equation using a weighting technique. He provided a formalism to calculate the value of the weighting parameter.

- The Burlin cavity theory can be written in its simplest form as follows:

$$\frac{D_{\text{gas}}}{D_{\text{med}}} = d\ s_{\text{gas,med}} + (1-d)\left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{gas,med}}, \tag{2.40}$$

where

| | |
|---|---|
| $d$ | is a parameter related to cavity size approaching unity for small cavities and zero for large ones; |
| $s_{\text{gas,med}}$ | is the mean ratio of the restricted mass stopping powers of the cavity and the medium; |
| $D_{\text{gas}}$ | is the absorbed dose in the cavity and |
| $(\bar{\mu}_{\text{en}}/\rho)_{\text{gas,med}}$ | is the mean ratio of the mass-energy absorption coefficients for the cavity and the medium. |

- The Burlin theory effectively requires that:

(1) The surrounding medium and the cavity medium are homogeneous;
(2) A homogeneous photon field exists everywhere throughout the medium and the cavity;
(3) Charged particle equilibrium exists at all points in the medium and the cavity that are further than the maximum electron range from the cavity boundary;
(4) The equilibrium spectra of secondary electrons generated in the medium and the cavity are the same.

- Burlin provided a method for estimating the weighting parameter $d$ in his theory. It is expressed as the average value of the electron fluence reduction in the medium. Consistent with experiments with β-sources he proposed that the electron fluence in the medium $\Phi^{e}_{\text{med}}$, on average, decays exponentially. The value of the weighting parameter $d$ in conjunction with the stopping power ratio can be calculated as:

$$d = \frac{\int_0^L \Phi^{e}_{\text{med}} e^{-\beta l} dl}{\int_0^L \Phi^{e}_{\text{med}} dl} = \frac{1-e^{-\beta L}}{\beta L}, \tag{2.41}$$

where β is an effective electron fluence attenuation coefficient that quantifies the reduction in particle fluence from its initial medium fluence value through a cavity of average length L. For convex cavities and isotropic electron fluence distributions, $L$ can be calculated as $4V/S$ where $V$ is the cavity volume and $S$ its surface area. Burlin described the build-up of the electron fluence $\Phi^e_{\text{gas}}$ inside the cavity using a similar, complementary equation:

$$1-d=\frac{\int_0^L \Phi^e_{\text{gas}}(1-e^{-\beta l})dl}{\int_0^L \Phi^e_{\text{gas}}dl}=\frac{\beta L-1+e^{-\beta L}}{\beta L}. \tag{2.42}$$

- Burlin's theory is consistent with the fundamental constraint of cavity theory that, the weighting factors of both terms add up to unity (*i.e.*, $d$ and 1-$d$). It had relative success in calculating ratios of absorbed dose for some types of intermediate cavities. More generally, however, Monte Carlo calculations show that, when studying ratios of directly calculated absorbed doses in the cavity to absorbed dose in the medium as a function of cavity size, the weighting method is too simplistic and additional terms are necessary to calculate dose ratios for intermediate cavity sizes. For these and other reasons, the Burlin cavity theory is no longer used in practice.

### 2.8.6. Stopping power ratios

- Although cavity theory was designed to calculate ratios of absorbed doses, the practical application of the Spencer-Attix cavity theory has always required additional correction factors. Since the central component of Spencer-Attix cavity theory results in averaging stopping powers, Spencer-Attix dose ratios are often referred to as "stopping power ratios".

- Stopping power ratios not only play a role in the absolute measurement of absorbed dose, they are also relevant in performing accurate relative measurements of absorbed dose in regimes where the energy of the secondary electrons changes significantly from one point in a phantom to another. An important example of this is apparent from Fig. 2.4 which shows restricted stopping power ratios ($\Delta$ = 10 keV) of water to air for electron beams as a function of depth in water. Note that these curves are for mono-energetic electrons; protocols or codes of practice for electron dosimetry provide fits of stopping power ratios for realistic accelerator beams. However, Fig. 2.4 shows clearly that the accurate measurement of electron beam depth dose curves requires depth-dependent correction factors.

- In photon beams, except at or near the surface, average restricted stopping power ratios of water to air do not vary significantly as a function of depth. Stopping power ratios (with $\Delta$ = 10 keV) under full build-up conditions are given in Table 2.1.

- More detailed information on stopping power ratios is given in Section 9.5.

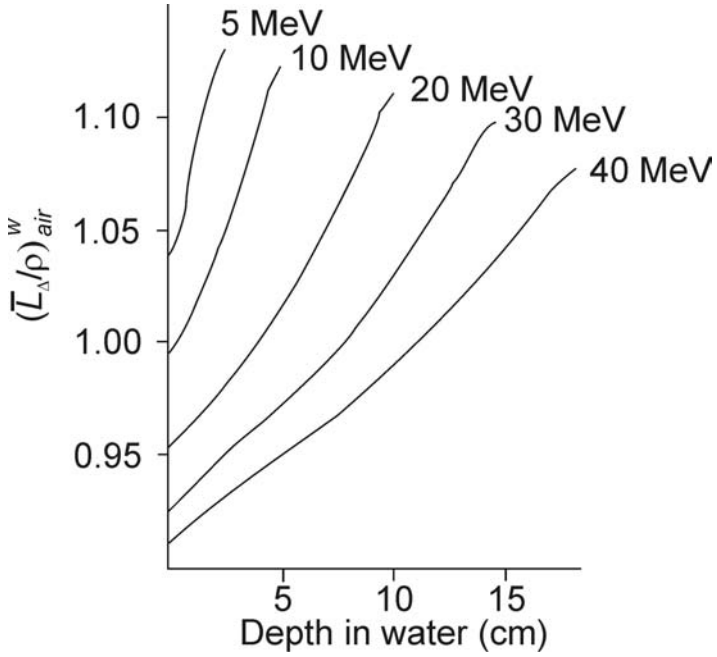


*FIG. 2.4. Restricted collision stopping power ratio of water to air ($\Delta = 10$ keV) as a function of depth for different mono-energetic electron energies.*

TABLE 2.I. AVERAGE RESTRICTED STOPPING POWER RATIO OF WATER TO AIR FOR DIFFERENT PHOTON SPECTRA IN THE RANGE FROM COBALT-60 GAMMA RAYS TO 35 MV X RAYS.

| Photon Spectra | $s_{water,air}$ |
|---|---|
| $^{60}$Co | 1.134 |
| 4 MV | 1.131 |
| 6 MV | 1.127 |
| 8 MV | 1.121 |
| 10 MV | 1.117 |
| 15 MV | 1.106 |
| 20 MV | 1.096 |
| 25 MV | 1.093 |
| 35 MV | 1.084 |

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